ENERGY RATIO OF THE SEISMIC WAVES REFLECTED AND REFRACTED AT A ROCK-WATER BOUNDARY*

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Introduction

The subject of energy ratios of seismic waves reflected and refracted at a discontinuity has been studied by various authors; a list of references is given at the end of this paper. They deal mostly with the theory and computations concerning the cases where a wave is incident at a boundary between two solid media. Reflection at the surface of the earth has been studied by Zoeppritz, Geiger, and Gutenberg,[1] Geiger and Gutenberg,[2] Gutenberg,[3] and Jeffreys.[4] The case of the mantle-core boundary has been computed by Dana.[5] The present paper deals with the case of the ocean floor. We are particularly interested in zero points and extreme points of energy ratios of waves resulting from the incidence of a wave at either side of the discontinuity.

Since this work was begun a few years ago, much progress has been accomplished in the study of microseisms and their use as a hurricane-detecting device. In this connection we hope that the results obtained in this paper may provide material for further studies concerning the passage of a wave, set up in the water by a storm, to the solid crustal layers.

SYMBOLS USED

P, longitudinal wave
S, transverse wave
SV, component of S in the plane of propagation
SH, component of S perpendicular to the plane of propagation

Incident		*Reflected*		*Refracted*		
P	S	P	S	P	S	
A	B	C	D	E	F	amplitudes
		c	d	e	f	square root of energy ratio
α	β	α	β	η	ζ	angle of incidence
		V_{1p}	V_{1s}	V_{2p}	V_{2s}	velocity
		ρ_1		ρ_2		density

σ, Poisson's ratio
$m = V_{1p}/V_{1s} = f(\sigma)$
$n = V_{2p}/V_{1p}$
$r = \rho_2/\rho_1$

Equations Used for Computation

Snell's law:

$$\sin \alpha : \sin \beta : \sin \eta : \sin \zeta = V_{1p} : V_{1s} : V_{2p} : V_{2s} \tag{1}$$

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[1] L. Zoeppritz, L. Geiger, and B. Gutenberg, "Über Erdbebenwellen. V," *Nachr. Gesell. d. Wiss. Göttingen*, math.-phys. Kl., 1912, pp. 121–206.

[2] L. Geiger and B. Gutenberg, "Über Erdbebenwellen. VI," *Nachr. Gesell. d. Wiss. Göttingen*, math.-phys. Kl., 1912, pp. 623–675.

[3] B. Gutenberg, "Energy Ratio of Reflected and Refracted Waves," *Bull. Seism. Soc. Am.*, 34: 85–102 (1944).

[4] H. Jeffreys, "The Reflection and Refraction of Elastic Waves," *Mon. Not. Roy. Astron. Soc., Geophys. Suppl.*, 1: 321–334 (1926).

[5] S. W. Dana, "The Partition of Energy among Seismic Waves Reflected and Refracted at the Earth's Core," *Bul. Seism. Soc. Am.*, 34: 189–198 (1944).

Blut's energy equations:

P wave incident in the solid against the water:

$$\frac{C^2}{A^2} + \frac{D^2 \sin 2\beta}{A^2 \sin 2\alpha} + \frac{E^2 \rho_2 \sin 2\eta}{A^2 \rho_1 \sin 2\alpha} = 1 \tag{2}$$

or

$$c^2 + d^2 + e^2 = 1$$

P wave incident in the water against the solid:

$$\frac{C^2}{A^2} + \frac{E^2 \rho_2 \sin 2\eta}{A^2 p_1 \sin 2\alpha} + \frac{F^2 \rho_2 \sin 2\zeta}{A^2 p_1 \sin 2\alpha} = 1 \tag{3}$$

or

$$c^2 + e^2 + f^2 = 1$$

SV wave incident in the solid against the water:

$$\frac{D^2}{B^2} + \frac{C^2 \sin 2\alpha}{B^2 \sin 2\beta} + \frac{E^2 \rho_2 \sin 2\eta}{B^2 \rho_1 \sin 2\beta} = 1 \tag{4}$$

or

$$d^2 + c^2 + e^2 = 1$$

Relation between σ and m

$$\sigma = \frac{1}{2}\left(1 - \frac{1}{m^2 - 1}\right). \tag{5}$$

The amplitude ratios $\frac{C}{A}, \frac{D}{A}, \frac{E}{A}$ and $\frac{F}{A}$ are computed from the following Zoeppritz equations:

P wave incident in the solid against the water:

$$A \cos \alpha - C \cos \alpha + D \sin \beta - E \cos \eta = 0 \tag{6a}$$

$$-A \cos 2\beta - C \cos 2\beta + D/m_1 \sin 2\beta + Enr = 0 \tag{6b}$$

$$-A \sin 2\alpha + C \sin 2\alpha + Dm_1 \cos 2\beta = 0 \tag{6c}$$

Solving these three equations and making use of equation (2), and also making use of the identity

$$m_1 \cos 2\beta \cos \alpha - \sin 2\alpha \sin \beta = m_1 \cos \alpha$$

we get for the square root of the energy ratios c, d, and e:

$$c = \frac{\cos \eta \,(m_1 \cos^2 2\beta - 1/m_1 \sin 2\alpha \sin 2\beta) - m_1 nr \cos \alpha}{\cos \eta \,(m_1 \cos^2 2\beta + 1/m_1 \sin 2\alpha \sin 2\beta) + m_1 nr \cos \alpha} \tag{7}$$

$$d = \frac{2\sqrt{\sin 2\alpha \sin 2\beta}\, \cos 2\beta \cos \eta}{\cos \eta \,(m_1 \cos^2 2\beta + 1/m_1 \sin 2\alpha \sin 2\beta) + m_1 nr \cos \alpha} \tag{8}$$

$$e = \frac{2m_1 \sqrt{nr \cos\alpha \cos\eta} \cos 2\beta}{\cos\eta\,(m_1 \cos^2 2\beta + 1/m_1 \sin 2\alpha \sin 2\beta) + m_1 nr \cos\alpha} \tag{9}$$

P wave incident in the water against the solid:

$$A \cos\alpha - C\cos\alpha - E\cos\eta - F\sin\zeta = 0 \tag{10a}$$

$$-A - C + Enr\cos 2\zeta + F\frac{rn}{m_2}\sin 2\zeta = 0 \tag{10b}$$

$$+ E\sin 2\eta - Fm_2\cos 2\zeta = 0 \tag{10c}$$

Solving these three equations and making use of equation (3), we get for the square root of the energy ratios c, e, and f:

$$c = \frac{\cos\eta - nr\cos\alpha\,[1 - 2\sin\zeta\sin 2\zeta\,(\cos\zeta - 1/m_2\cos\eta)]}{\cos\eta + nr\cos\alpha\,[1 - 2\sin\zeta\sin 2\zeta\,(\cos\zeta - 1/m_2\cos\eta)]} \tag{11}$$

$$e = \frac{2\sqrt{nr\cos\alpha\cos\eta}\cos 2\zeta}{\cos\eta + nr\cos\alpha\,[1 - 2\sin\zeta\sin 2\zeta\,(\cos\zeta - 1/m_2\cos\eta)]} \tag{12}$$

$$f = \frac{2n/m_2\sqrt{r\sin 2\alpha\sin 2\zeta}\cos\eta}{\cos\eta + nr\cos\alpha\,[1 - 2\sin\zeta\sin 2\zeta\,(\cos\zeta - 1/m_2\cos\eta)]} \tag{13}$$

SV wave incident in the solid against the water. (Note that in this case angle of incidence is β):

$$-B\sin\beta - C\cos\alpha + D\sin\beta - E\cos\eta = 0 \tag{14a}$$

$$B\sin 2\beta - Cm_1\cos 2\beta + D\sin 2\beta + Em_1nr = 0 \tag{14b}$$

$$-B\cos 2\beta + C\,1/m_1\sin 2\alpha + D\cos 2\beta = 0 \tag{14c}$$

Solving these three equations and making use of equation (4), and again making use of the identity $m_1 \cos 2\beta \cos\alpha - \sin 2\alpha \sin\beta = m_1\cos\alpha$, we get for the square root of the energy ratios c, d and f:

$$d = \frac{\cos\eta\,(m_1\cos^2 2\beta - 1/m_1\sin 2\alpha\sin 2\beta) + m_1nr\cos\alpha}{\cos\eta\,(m_1\cos^2 2\beta + 1/m_1\sin 2\alpha\sin 2\beta) + m_1nr\cos\alpha} \tag{15}$$

$$c = \frac{2\sqrt{\sin 2\alpha\sin 2\beta}\cos 2\beta\cos\eta}{\cos\eta\,(m_1\cos^2 2\beta + 1/m_1\sin 2\alpha\sin 2\beta) + m_1nr\cos\alpha} \tag{16}$$

$$e = \frac{2\sqrt{r\sin 2\beta\sin 2\eta}\cos\alpha}{\cos\eta\,(m_1\cos^2 2\beta + 1/m_1\sin 2\alpha\sin 2\beta) + m_1nr\cos\alpha} \tag{17}$$

SH wave incident in the solid against the water. For this particular case Zoeppritz equations reduce to

$$B = D$$

so that all of the energy is reflected as SH wave for all angles of incidence.

Extreme Points of $\sqrt{E_{\text{ref p}} / E_{\text{inc p}}}$ and $\sqrt{E_{\text{ref s}} / E_{\text{inc s}}}$

Differentiating (7) with respect to β and putting $\frac{\partial c}{\partial \beta} = 0$ we get

$$2(1 - m_1^2) \sin^4 \beta + (3m_1^2 - 1) \sin^2 \beta - 2$$

$$+ \frac{nr}{1 - 2 \sin^2 \beta} (1 - m_1^2 \sin^2 \beta)^{3/2} (1 - \sin^2 \beta) = 0 \tag{7a}$$

The value of β that satisfies this equation is the angle of incidence of the reflected S wave at which the reflected P wave has its extreme value. The corresponding value of α can be computed from Snell's law

$$\sin \alpha : \sin \beta = V_{1p}/V_{1s}$$

When a P wave is incident at the surface of the earth, then $n = 0$, and the following equation gives the extreme points of the energy ratio of the P wave reflected at the surface of the earth,

$$2(1 - m_1^2) \sin^4 \beta + (3m_1^2 - 1) \sin^2 \beta - 2 = 0 \tag{7b}$$

Last term of the equation (7a) is very small compared to the rest of the terms. The same statement is also true for the equation (7). This fact can be used to explain the similarity in the behavior of the reflected wave that is of the same type as the incident wave, in both cases, i.e., the rock-water boundary and the surface of the earth.

In the case of an SV wave incident in the rock against the water, we get a similar expression for the extreme points of the reflected SV wave, the only difference being that the sign of the last term is negative.

$$2(1 - m_1^2) \sin^4 \beta + (3m_1^2 - 1) \sin^2 \beta - 2$$

$$- \frac{nr}{1 - 2 \sin^2 \beta} (1 - m_1^2 \sin^2 \beta)^{3/2} (1 - \sin^2 \beta) = 0 \tag{15a}$$

When an SV wave is incident at the surface of the earth we again get the equation (7b). This result should be expected, because for the surface of the earth the expression giving $\sqrt{E_{\text{ref p}} / E_{\text{inc p}}}$ is the same as the expression giving $\sqrt{E_{\text{ref s}} / E_{\text{inc s}}}$. (See Gutenberg, "Theorie der Erdbebenwellen.")

Computed Values

In general, computation falls into three groups:

1) The general behavior of the energy ratio for each case involves calculation of the square root of the energy ratio for a sufficient number of the angles of incidence. This was done with a slide rule to the third decimal place, using one of the equations (7), (8), (9), (11), (12), (13), (15), (16), and (17), as the case might be.

2) The angle of incidence (if any) at which an energy ratio becomes zero was determined, first, by determining this angle from the graphs showing the general behavior of the energy ratio, approximately, and then by computing more values with a calculating machine to the fourth decimal place in the neighborhood of the approximate value. The final value of the zero point was thus determined graphically.

3) The angle of incidence (if any) at which an energy ratio has an extreme value was also determined graphically.

The computation is carried out for possible values of three parameters, namely, m, n, and r. Results are tabulated and plotted against the angle of incidence and are given in the following pages. For the general case, according to Gutenberg, "the results of calculations are in good agreement with the observations. However, for a wave which travels almost tangent to the discontinuity and in the medium with higher velocity, the calculated energy may be much too small."[6] The same situation should be true for the special cases considered here.

P WAVE INCIDENT IN THE SOLID AGAINST THE WATER

Using equations (7), (8), and (9), energy ratios are computed for the following values of parameters: $m_1 = 1.6, 1.7, 1.8$; $n = 0.2, 0.3, 0.4$; $r = 0.3, 0.4, 0.5$.

For different values of parameters, values of c as a function of α are given in table 1, values of α_0 (α for which $c = 0$) in table 2, and values of α_{ext} (α for which c has an extreme value) in table 3. Figure 1 shows the general behavior of c. In order that the effect of each parameter should be shown, two of them were kept constant and the energy ratios were computed for three different values of the third parameter (m_1 is varied for two sets of values of n and r). From these computations it is found that the effect of changing m_1 is greater than the effect of changing either n or r. In figure 2, α_{01}, α_{02}, and α_{ext} are plotted against m_1 for different values of n and r. Here we note that for all values of n and r, α approaches 90° as m_1 approaches $\sqrt{2}$ ($\sigma = 0$). From these curves it is evident that theoretically the α_0 curve intersects the α_{ext} curve which means that there are some combinations of values of parameters for which the extreme value of c is zero. The following two examples are given to illustrate the possibility of such an incidence for actual values of densities and elastic constants.

[6] As cited in note 3 above. He adds references to G. Joos and J. Teltow, "Zur Deutung der Knallwellenausbreitung und der Trennschicht zweier Medien," *Phys. Zeitschr.*, 40: 289–293 (1930), and H. Ott, "Reflexion und Brechung von Kugelwellen," *Ann. d. Physik*, 38: 443–466 (1942).

TABLE 1

INCIDENT P WAVE IN THE SOLID AGAINST THE WATER: SQUARE ROOT OF THE ENERGY RATIO OF THE REFLECTED P WAVE

(For the purpose of showing the change of sign at zero points, the energy ratio is tabulated with the sign of the amplitude ratio.)

m		1.6			1.7			1.8		
r		0.3	0.4	0.5	0.3	0.4	0.5	0.3	0.4	0.5
n	α (deg.)									
0.2	0	−0.887	−0.852	−0.818	−0.887	−0.852	−0.818	−0.887	−0.852	−0.818
	10	− .831	− .797	− .764	− .840	− .806	− .773	− .850	− .816	− .783
	20	− .672	− .641	− .611	− .708	− .676	− .645	− .736	− .705	− .673
	30	− .439	− .411	− .385	− .513	− .484	− .457	− .571	− .542	− .514
	40	− .163	− .141	− .119	− .283	− .259	− .235	− .378	− .352	− .327
	50	+ .106	+ .123	+ .140	− .057	− .038	− .018	− .195	− .172	− .150
	60	+ .327	+ .340	+ .353	+ .115	+ .137	+ .148	− .062	− .043	− .024
	70	+ .462	+ .473	+ .483	+ .197	+ .212	+ .225	− .013	+ .001	+ .019
	80	+ .406	+ .415	+ .426	+ .060	+ .074	+ .088	− .193	− .178	− .164
	90	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000
0.4	0	−0.786	−0.724	−0.667	−0.786	−0.724	−0.667	−0.786	−0.724	−0.667
	10	− .732	− .672	− .616	− .741	− .680	− .624	− .749	− .690	− .634
	20	− .580	− .524	− .473	− .614	− .558	− .505	− .636	− .584	− .531
	30	− .357	− .308	− .264	− .428	− .377	− .330	− .475	− .434	− .380
	40	− .096	− .056	− .019	− .209	− .164	− .123	− .292	− .250	− .208
	50	+ .159	+ .191	+ .220	+ .005	+ .014	+ .075	− .120	− .085	− .047
	60	+ .369	+ .393	+ .415	+ .170	+ .201	+ .229	− .001	+ .034	+ .068
	70	+ .496	+ .515	+ .532	+ .244	+ .270	+ .295	+ .030	+ .063	+ .103
	80	+ .441	+ .461	+ .479	+ .105	+ .133	+ .158	+ .149	+ .117	+ .082
	90	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000

TABLE 2

INCIDENT P WAVE IN THE SOLID AGAINST THE WATER: ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFLECTED P WAVE IS ZERO

r		0.3		0.4		0.5	
n	m	α_{01}	α_{02}	α_{01}	α_{02}	α_{01}	α_{02}
		deg. min.	deg. min.	deg. min.	deg. min.	deg. min.	deg. min.
0.2	1.60	45 58	86 32	45 10	86 38	44 30	86 45
	1.65	49 11	84 27	48 20	84 30	47 30	84 40
	1.70	52 48	81 32	52 00	81 50	51 04	82 08
	1.75	58 03	77 19	56 40	77 52	55 28	78 23
	1.785	64.5	72.0				
0.3	1.60	44 46	86 41	43 45	86 51	42 38	86 57
	1.65	47 49	84 42	46 36	84 55	45 41	85 07
	1.70	51 25	82 02	50 08	82 13	48 33	82 41
	1.75	56 04	78 21	54 05	78 59	52 24	79 35
0.4	1.60	43 34	86 40	42 12	86 59	40 52	87 07
	1.65	46 34	85 20	44 44	85 22	43 15	85 27
	1.70	49 49	82 27	47 50	83 02	46 13	83 16
	1.75	53 58	78 54	51 38	79 52	49 28	80 40
	1.80	59 41	74.0	56 16	75 45	53 28	77 03

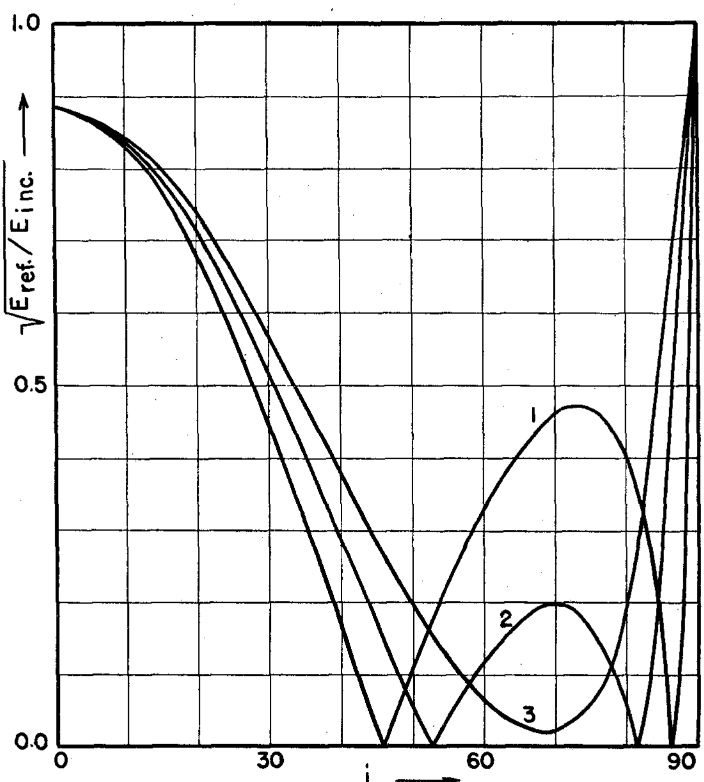

Fig. 1. P wave incident in the solid against the water. Square root of the energy ratio of the reflected P wave.

Curve	V_{1p}/V_{1s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.8	0.2	0.3

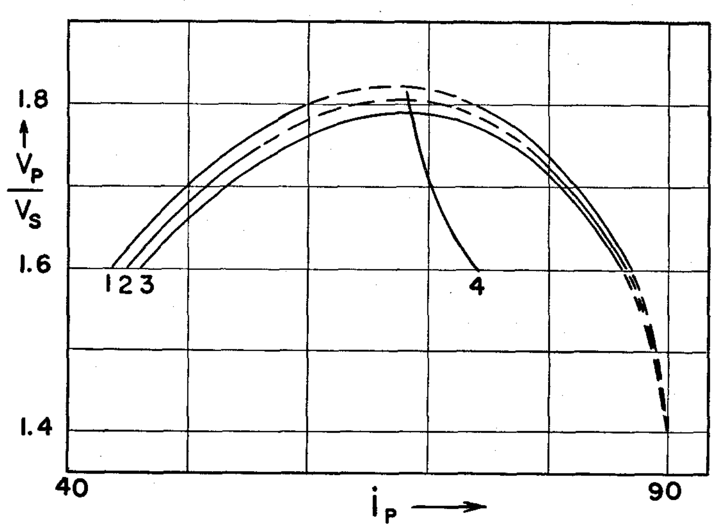

Fig. 2. P wave incident in the solid against the water.

Curve		V_{2p}/V_{1p}	ρ_2/ρ_1
1	zeros of the energy ratio of the reflected P wave	0.4	0.3
2		0.3	0.3
3		0.2	0.3
4	Extreme points of the energy ratio of the reflected P wave	0.2	0.3

Example 1. Granite–sea water boundary:

$V_{1p} = 5.55$	$V_{1s} = 3.26$	$V_{2p} = 1.5$	$\rho_2 = 1.026$	$\rho_1 = 2.9$
Hence	$\sigma = 0.24$	$m_1 = 1.70$	$n = 0.27$	$r = 0.354$

These values of parameters are represented by the curves and corresponding values of α_{01}, α_{02} and α_{ext} can easily be obtained by interpolation. It is evident that for this example c has two distinct roots and one extreme value between them.

Example 2. Ultrabasic rock–sea water boundary:

$V_{1p} = 8.0$	$V_{1s} = 4.4$	$V_{2p} = 1.5$	$\rho_2 = 1.026$	$\rho_1 = 3.3$
Hence	$\sigma = 0.27$	$m_1 = 1.82$	$n = 0.1875$	$r = 0.311$

From the curves it is evident that for these values of parameters c has no roots, but it has an extreme value. Thus, we conclude that for a solid medium with seismic velocities and density lying between those for granite and an ultrabasic rock such as dunite, c may have an extreme point which is at the same time a double root of the equation.

In table 4, the values of d and α_{ext} for d are given for different values of parameters. The values of d are plotted in figure 3. Changes in r produce very little change in d. d starts from zero at $\alpha = 0°$ increases rapidly. For small values of m_1 it has two distinct maxima and a minimum in-between. As m_1 increases the extreme points become less pronounced and for $m_1 = 1.8$ they are represented by a flat section between $\alpha = 60°$ and $\alpha = 80°$. α_{ext} are not affected very much as we vary n and r.

Table 5 and figure 4 show the values of e for different values of parameters. e starts with a value $\dfrac{2\sqrt{nr}}{1 + nr}$ at $\alpha = 0°$; as α increases, it diminishes very slowly, but in the neighborhood of $\alpha = 90°$ very sharply and becomes zero at $\alpha = 90°$. For smaller m_1 it shows a slight increase after $\alpha \sim 75°$ and around $\alpha \sim 87$ it starts decreasing sharply to zero at $\alpha = 90°$. As m_1 takes larger values this tendency seems to disappear. Changes in n and r have negligible effect on these turning points. For larger values of n, e has larger values at all angles of incidence. This simply means that the less longitudinal velocity contrast we have, the more energy goes into the refracted P wave.

P WAVE INCIDENT IN THE WATER AGAINST THE SOLID

Using equations (11), (12), and (13), energy ratios are computed for the following values of parameters: $m_2 = 1.6, 1.7, 1.8$ ($\sigma = 0.18$–0.277); $n = 3.0, 4.0$; $r = 2.5, 3.0$.

For this particular case we have total reflections, occurring twice; namely, one at $\eta = 90°$ and one at $\zeta = 90°$. We shall call corresponding angles of incidence $\alpha_{c\eta}$ and $\alpha_{c\zeta}$ respectively in the following discussions. At these critical incidences all of the energy is reflected as P wave.

Values of c, e, and f are given in tables 6, 7, and 8, respectively; those for α_{ext}, in table 9. These results are plotted in figures 5 and 6.

c starts with a value $\dfrac{1 - rn}{1 + rn}$ at $\alpha = 0°$, staying almost constant until near $\alpha_{c\eta}$. As $\alpha_{c\eta}$ is approached it rapidly increases and becomes *one* at $\alpha_{c\eta}$. Right after, it suddenly drops down to a value less than it has for $\alpha < \alpha_{c\eta}$, then it starts increasing slowly, the rate of increase diminishing gradually, and as it gets close to $\alpha_{c\zeta}$ it starts

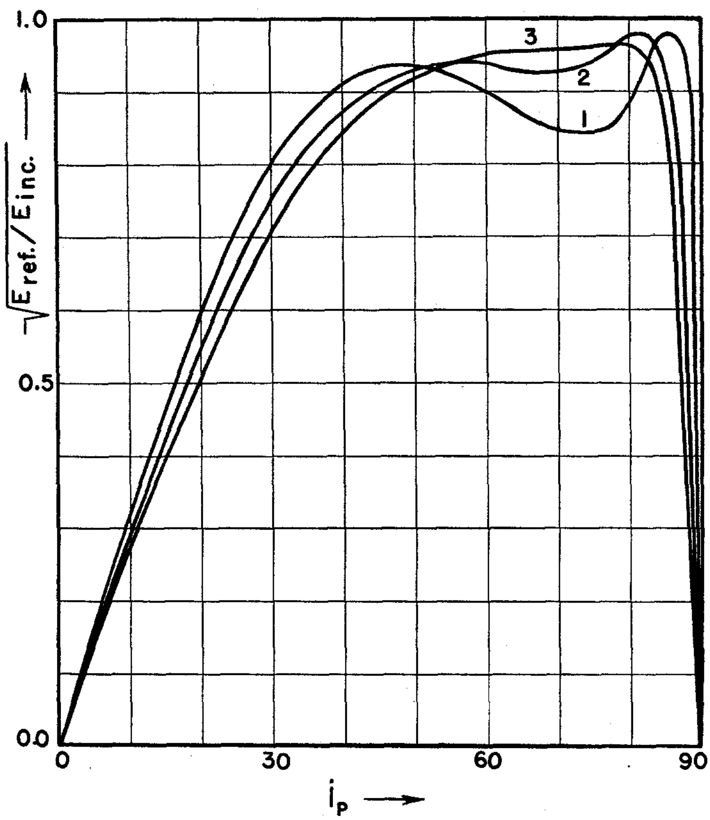

Fig. 3. P wave incident in the solid against the water. Square root of the energy ratio of the reflected SV wave.

Curve	V_{1p}/V_{1s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.8	0.2	0.3

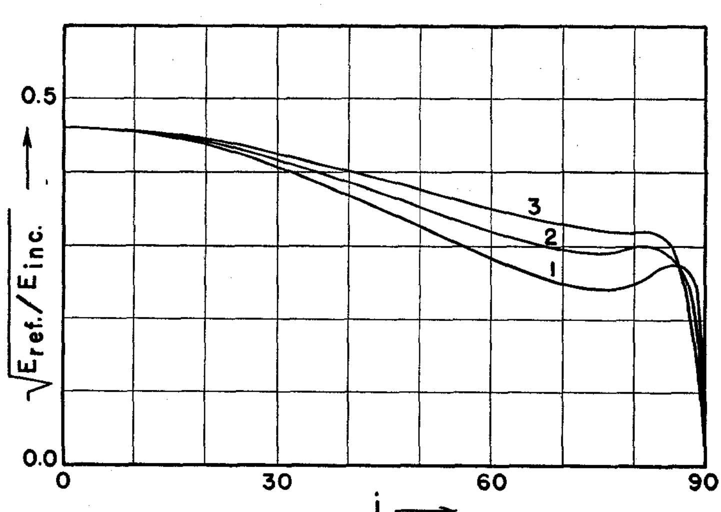

Fig. 4. P wave incident in the solid against the water. Square root of the energy ratio of the refracted P wave.

Curve	V_{1p}/V_{1s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.8	0.2	0.3

TABLE 3

INCIDENT P WAVE IN THE SOLID AGAINST THE WATER: ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFLECTED P WAVE HAS ITS EXTREME VALUE
(For the purpose of showing the change of sign at zero points, the energy ratio is tabulated with the sign of the amplitude ratio.)

n	0.2		0.2	0.4
r	0.3		0.6	0.3
m_1	α_{ext}	c	α_{ext}	α_{ext}
	deg. min.		deg. min.	deg. min.
1.60	74 00	+0.4725		
1.65	71 41	+0.3280		
1.70	70 08	+0.1973		
1.75	68 50	+0.0830	68 45	68 36
1.79	68 25	+0.000		
1.80	68 15	−0.0185		

TABLE 4

INCIDENT P WAVE IN THE SOLID AGAINST THE WATER
(*a*) THE SQUARE ROOT OF THE ENERGY RATIO OF THE REFLECTED S WAVE

n	0.2			0.4	0.2
r	0.3			0.3	0.4
m	1.6	1.7	1.8	1.7	1.7
α (deg.)					
0	0.000	0.000	0.000	0.000	0.000
10	.318	.289	.269	.273	.284
20	.600	.552	.508	.519	.541
30	.804	.748	.704	.705	.733
40	.914	.875	.841	.826	.868
50	.936	.930	.917	.878	.913
60	.900	.941	.954	.883	.923
70	.844	.925	.960	.869	.907
75	.844	.940	....	.885	.924
80	.882	.979	.967	.927	.965
85	.978	.929	.869	.892	.917
87	.970	....	....	....	....
90	0.000	0.000	0.000	0.000	0.000

TABLE 4 (*Continued*)

(*b*) ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFLECTED S WAVE HAS ITS EXTREME VALUE

n	0.2		
r	0.3		
m_1	α_{ext}	α_{ext}	α_{ext}
1.60	48°2	73°2	86°5
1.65	51.7	71.5	84.2
1.70	56.5	68.5	81.5
1.725	60	66	
1.75	....	....	76

TABLE 5

INCIDENT P WAVE IN THE SOLID AGAINST THE WATER: SQUARE ROOT OF THE ENERGY RATIO OF THE REFRACTED P WAVE

n	0.2			0.4	0.2
r	0.3			0.3	0.4
m_1	1.6	1.7	1.8	1.7	1.7
α (deg.)					
0	0.462	0.462	0.462	0.618	0.453
10	.455	.457	.458	.614	.448
20	.432	.440	.442	.593	.431
30	.415	.419	.426	.562	.411
40	.370	.387	.402	.517	.380
50	.325	.351	.373	.475	.345
60	.284	.321	.352	.433	.315
70	.250	.294	.330	.404	.288
75	.242	.292	.322	.400	.287
80	.251	.300	.319	.420	.295
85	.271	.279	.285	.406	.276
87	.272	.259	.237	....	....
90	0.000	0.000	0.000	0.000	0.000

increasing sharply and at $\alpha = \alpha_{c\xi}$, $c = 1$. Values of α at which the energy curve starts increasing right after $\alpha_{c\eta}$ are given in table 9.

e starts with a value $\dfrac{2\sqrt{nr}}{1 + nr}$ at $\alpha = 0°$, and decreases slowly until near $\alpha_{c\eta}$, where it shows little increase before it drops to zero at $\alpha_{c\eta}$. For small values of m_1, for instance, $m = 1.3$ ($\sigma < 0$, given here just as an illustration), it has a zero between $\alpha = 0°$ and $\alpha = \alpha_{c\eta}$, and for large values of m_1 it is a smooth, continuously decreasing function of α. After $\alpha_{c\eta}$ no refracted P wave exists. $e = 0$.

f starts from zero at $\alpha = 0$, and for a while increases almost linearly; as α approaches $\alpha_{c\eta}$, it turns down and decreases sharply to zero at $\alpha_{c\eta}$. Right after $\alpha_{c\eta}$ it increases suddenly and attains a value larger than it has for $\alpha < \alpha_{c\eta}$. Here it turns and decreases very slowly, the rate of decrease becoming smaller, and as $\alpha_{c\xi}$ is approached it decreases sharply and becomes zero at $\alpha_{c\xi}$. After $\alpha_{c\xi}$ there is no refracted S wave either. Thus all the energy goes into reflected P wave. Turning points occurring just before and right after $\alpha_{c\eta}$ are given in table 9.

It is evident that here, in this particular case, critical angles of incidences are the governing factors. All peculiarities occur just before and right after these angles.

Here we notice that most of the energy goes into the reflected P wave. None of the energy ratios becomes zero except at a critical incidence. The turning points which we may call α_{ext} are not very much different for different values of r, but they depend on m and n.

SV WAVE INCIDENT IN THE SOLID AGAINST THE WATER

Using equations (15), (16), and (17), energy ratios have been computed and their zeros and extreme points investigated for the following values of parameters: $m_1 = 1.6, 1.7, 1.75$ ($\sigma = 0.18 - 0.2576$), $n = 0.2, 0.3, 0.4$ (and β_{ext} were computed in addition to these values for $m = 1.8$ [$\sigma = 0.277$]); $r = 0.3$.

In this particular case there is only one total reflection at $\beta_{c\alpha}$ ($\alpha = 90°$). At this critical angle of incidence $d = 1$, $e = 0$, $c = 0$. The part of the curves for d and c that lies between $\beta = 0$ and $\beta = \beta_{c\alpha}$ are similar to the curves of c and d for the case of a P wave incident in the solid against the water. The curve for e as a whole is similar to that of f for the case of a P wave incident in the water against the solid, between $\alpha = 0$ and $\alpha = \alpha_{c\xi}$. The only difference between these similar curves is that, in the case of incident SV wave, at normal incidence all the energy is reflected as SV wave; in other cases, at normal incidence the energy is split between c and e.

Here again we notice that β_{ext} are not very much different for different values of n and r. Therefore for practical purposes β_{ext} may be considered as a function of m only.

The characteristics of the curves for SV incident in solid against water are given in tables 10–15 and figures 7–10.

TABLE 6

INCIDENT P WAVE IN THE WATER AGAINST THE SOLID: SQUARE ROOT OF THE ENERGY RATIO OF THE REFLECTED P WAVE

m_2		1.6		1.7		1.8	
r		2.5	3.0	2.5	3.0	2.5	3.0
n	a (deg. min. sec.)						
3.0	0	0.764	0.800	0.764	0.800	0.764	0.800
	5	.764	.799	.764	.799	.761	.798
	10	.763	.798	.760	.795	.760	.796
	15	.747	.787	.753	.790	.738	.794
	18	.749	.787	.762	.802	.774	.810
	19	.762	.797	.788	.823	.821	.848
	19 28 16	1.000	1.000	1.000	1.000	1.000	1.000
	25	.740	.775	.710	.753	.682	.728
	30	.761	.795	.723	.766	.702	.745
	31	.786	.820	.....	.....	.....	.....
	32 13 51	1.000	1.000	.....	.....	.....	.....
	33	.....	.....	.755	.790	.....	.....
	34	.....	.....	.....	.....	.....	.750
	34 31 05	.....	.....	1.000	1.000	.....	.....
	35	.....	.....	.....	.....	.725	.765
	36	.....	.....	.....	.....	.....	.808
	36 52 12	.....	.....	.....	.....	1.000	1.000
4.0	0	0.818	0.846	0.818	0.846	0.818	0.846
	5	.818	.845	.818	.844	.816	.844
	10	.812	.838	.812	.841	.816	.844
	13	.809	.836	.814	.844	.814	.851
	14	.814	.844	.832	.858	.846	.872
	14 28 39	1.000	1.000	1.000	1.000	1.000	1.000
	15	.771	.808	.768	.819	.792	.822
	20	.805	.837	.793	.821	.776	.810
	22	.782	.813	.....	.....	.....	.....
	23	.828	.870	.800	.793	.782	.816
	23 34 41	1.000	1.000	.....	.....	.....	.....
	25	.....	.....	.908	.920	.797	.825
	25 09 02	.....	.....	1.000	1.000	.....	.....
	26 44 36	.....	.....	.....	.....	1.000	1.000

TABLE 7

INCIDENT P WAVE IN THE WATER AGAINST THE SOLID: SQUARE ROOT OF THE ENERGY OF THE REFRACTED P WAVE

m_2		1.6		1.7		1.8	
r		2.5	3.0	2.5	3.0	2.5	3.0
n	a deg. min. sec.						
3.0	0	0.645	0.600	0.645	0.600	0.645	0.600
	5	.626	.584	.628	.584	.644	.587
	10	.572	.532	.579	.537	.589	.549
	15	.513	.424	.490	.457	.520	.485
	18	.405	.338	.420	.391	.480	.434
	18 30	....	.327	.427	.398	....	.430
	19	.352	.328	.420	.389	.465	.429
	19 28 16	0.000	0.000	0.000	0.000	0.000	0.000
4.0	0	0.575	0.532	0.575	0.532	0.575	0.532
	5	.543	.503	.549	.508	.553	.513
	10	.439	.407	.462	.428	.481	.448
	13	.332	.308	.359	.332	.450	.393
	14	.306	.284	.254	.340	.400	.370
	14 28 39	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 8

INCIDENT P WAVE IN THE WATER AGAINST THE SOLID: SQUARE ROOT OF THE ENERGY RATIO OF THE REFRACTED S WAVE

	m_2..............	1.6		1.7		1.8	
	r..............	2.5	3.0	2.5	3.0	2.5	3.0
n	α deg. min. sec.						
3.0	0	0.000	0.000	0.000	0.000	0.000	0.000
	5	.165	.151	.152	.142	.142	.130
	10	.314	.298	.305	.280	.274	.253
	15	.487	.455	.441	.410	.398	.373
	18	.557	.520	.485	.452	.427	.396
	19	.543	.503	.452	.420	.351	.326
	19 28 16	.000	.000	.000	.000	.000	.000
	25	.680	.637	.705	.662	.732	.690
	30	.648	.605	.688	.644	.695	.658
	31	.416	.387	.....	.....	.....	.....
	32 13 51	0.000	0.000	.....	.....	.....	.....
	33	.....	.....	.655	.613	.....	.....
	34	.....	.....	.....	.....	.....	.662
	34 31 05	.....	.....	0.000	0.000	.....	.....
	35	.....	.....	.....	.....	.705	.660
	36	.....	.....	.....	.....	.....	.598
	36 52 12	.....	.....	.....	.....	0.000	0.000
4.0	0	0.000	0.000	0.000	0.000	0.000	0.000
	5	.197	.183	.181	.168	.164	.153
	10	.391	.361	.345	.320	.323	.299
	13	.465	.430	.438	.407	.403	.352
	14	.488	.455	.417	.385	.351	.325
	14 28 39	.000	.000	.000	.000	.000	.000
	15	.630	.579	.550	.513	.611	.566
	20	.587	.546	.612	.566	.635	.590
	22	.623	.580	.....	.....	.....	.....
	23	.519	.492	.598	.555	.627	.583
	23 34 41	0.000	0.000	.....	.....	.....	.....
	25	.....	.....	.424	.390	.609	.565
	25 09 02	.....	.....	0.000	0.000	.....	.....
	26 44 36	.....	.....	.....	.....	0.000	0.000

TABLE 9

INCIDENT P WAVE IN THE WATER AGAINST THE SOLID: ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFRACTED S WAVE HAS ITS EXTREME VALUE

(*a*) FOR ANGLES OF INCIDENCE GREATER THAN THE CRITICAL ANGLE OF INCIDENCE

r..........	3.0		
n..........	3.0	3.5	4.0
m_2			
1.6	18°.3	16°.0	13°.7
1.7	18°.0	15°.5	13°.4
1.8	17°.2	15°.1	13°.0

(*b*) FOR ANGLES OF INCIDENCE LESS THAN THE CRITICAL ANGLE OF INCIDENCE

r..........	3.0		
n..........	3.0	3.5	4.0
m_2			
1.6	20°.3	17°.2	15°.0
1.7	20°.9	17°.8	15°.4
1.8	21°.8	18°.6	16°.0

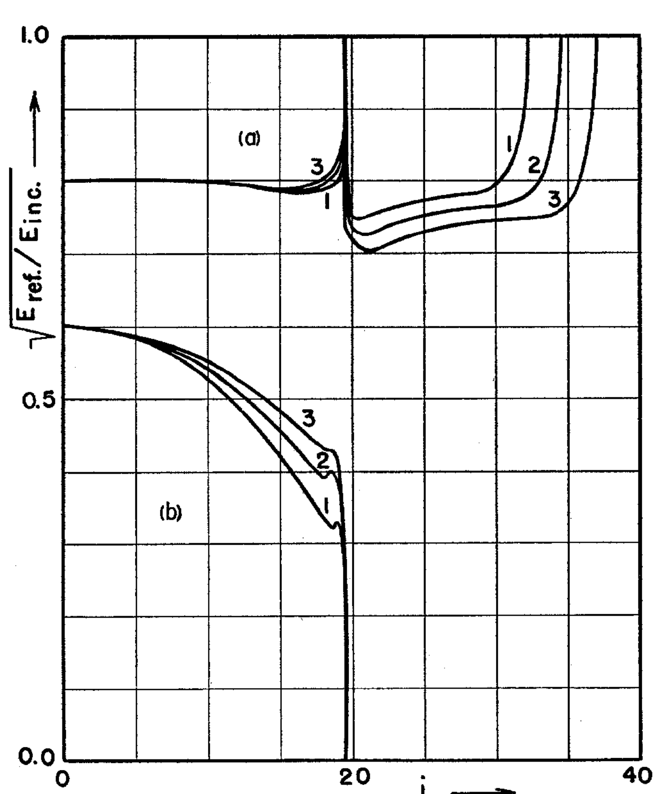

Fig. 5. P wave incident in the water against the solid. (a) Square root of the energy ratio of the reflected P wave. (b) Square root of the energy ratio of the refracted P wave.

Curve	V_{2p}/V_{2s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	3.0	3.0
2	1.7	3.0	3.0
3	1.8	3.0	3.0

TABLE 10

INCIDENT SV WAVE IN THE SOLID AGAINST THE WATER: SQUARE ROOT OF THE ENERGY RATIO OF THE REFLECTED S WAVE

(For the purpose of showing the change of sign at zero points, the energy ratio is tabulated with the sign of the amplitude ratio.)

n	0.2			0.2	0.4
r	0.3			0.4	0.3
m_1	1.6	1.7	1-75	1.7	1.7
β deg. min. sec.					
0	−1.000	−1.000	−1.000	−1.000	−1.000
10	− .859	− .867	− .871	− .870	− .874
20	− .470	− .503	− .522	− .512	− .530
30	+ .061	− .025	− .077	− .044	− .085
34	.....	+ .086	.....	+ .067	+ .024
34 51 00	.....	.....	−1.000	.....	.....
35	.....	.....	− .922	.....	.....
36 01 55	.....	−1.000	.....	−1.000	−1.000
37	.....	− .872	.....	− .844	− .749
38 40 56	−1.000	.....	.....	.....	.....
40	− .864	− .851	− .845	− .804	− .702
50	− .878	− .869	− .867	− .829	− .727
60	− .886	− .875	− .875	− .836	− .736
70	− .895	− .882	− .880	− .848	− .744
80	− .927	− .922	− .915	− .904	− .818
90	−1.000	−1.000	−1.000	−1.000	−1.000

TABLE 11

INCIDENT SV WAVE IN THE SOLID AGAINST THE WATER: ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFLECTED S WAVE IS ZERO

n		0.2		0.3		0.4	
r	m_1	β_{01}	β_{02}	β_{01}	β_{02}	β_{01}	β_{02}
0.3	1.60	28°51′	38°33′.5	29°25′	38°32′.2	30°02′	38°50′.9
	1.65	29 36	36 59.1	30 12	36 55.7	30 51	36 51.6
	1.70	30 35	35 15	31 19	35 05.8	32 15	34 55.1
	1.725	31 26	34 05	32 46	33 23		
0.4	1.60	29 13	38 32′.8	29 57	38 30′.3	30 50	38 29′.1
	1.65	30 01	36 57.1	30 48	36 52.2	31 44	36 46.3
	1.70	31 03	35 11.2	32 04	34 55.5	33 22	34 40.5
	1.725	32 15	33 50				

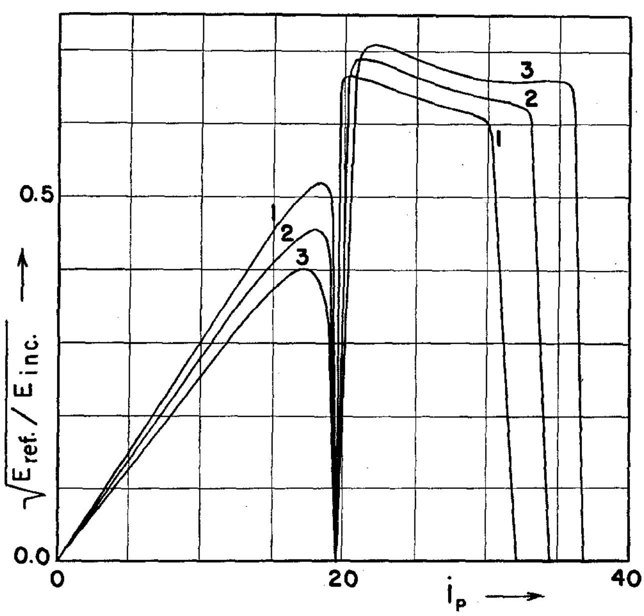

Fig. 6. P wave incident in the water against the solid. Square root of the energy ratio of the refracted SV wave.

Curve	V_{2p}/V_{2s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	3.0	3.0
2	1.7	3.0	3.0
3	1.8	3.0	3.0

TABLE 12

INCIDENT SV WAVE IN THE SOLID AGAINST THE WATER: ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFLECTED S WAVE HAS ITS EXTREME VALUE

(For the purpose of showing the change of sign at zero points, the energy ratio has been entered in the third column, with the sign of the amplitude ratio.)

n	0.2		0.3			0.4		
r	0.3		0.2	0.3	0.4	0.2	0.3	0.4
m_1	β_{ext}	d	β_{ext}	β_{ext}	β_{ext}	β_{ext}	β_{ext}	β_{ext}
1.60	37°00′	+.3572	36°56′	36°58′	37°00′	36°55′	36°57′	36°58′
1.65	35 10	+.2142	35 16	35 17	35 18			
1.70	33 40	+.0880	33 43	33 44	33 45			
1.7375	32 30	+.0000						
1.75	32 10	−.0250	32 19	32 20	32 21			
1.80	30 55	−.1350	31 06	31 07	31 08			

TABLE 13

INCIDENT SV WAVE IN THE SOLID AGAINST THE WATER: SQUARE ROOT OF THE ENERGY RATIO OF THE REFLECTED P WAVE

n	0.2			0.2	0.4
r	0.3			0.4	0.3
m_1	1.6	1.7	1.75	1.7	1.7
β deg. min. sec.					
0	0.000	0.000	0.000	0.000	0.000
10	.497	.482	.475	.472	.456
20	.847	.831	.822	.814	.783
25	.928	.915	.908	....	....
28	.939	.932	.935	....	....
30	.933	.930	.943	.912	.883
32	.909	.938	.929	....	....
34	....	....	....	.920	.898
34 51 00	....	....	0.000	....	....
35	.846	.948	....	....	....
36	.854	....	....	....	....
36 01 55	....	0.000	....	0.000	0.000
37	.857	....	....	....	....
38	.887	....	....	....	....
38 40 56	0.000	....	....	....	....

TABLE 14

INCIDENT SV WAVE IN THE SOLID AGAINST THE WATER: SQUARE ROOT OF THE ENERGY RATIO OF THE REFRACTED P WAVE

n	0.2			0.2	0.4
r	0.3			0.4	0.3
m_1	1.6	1.7	1.75	1.7	1.7
β deg. min. sec.					
0	0.000	0.000	0.000	0.000	0.000
10	..128	.122	.120	.139	.191
20	.259	.241	.239	.272	.373
30	.357	.338	.324	.382	.535
34	....	.343	....	.390	.555
34 51 00	....	....	.000	....	....
35	....	....	.250	....	....
36 01 55	....	.000	....	.000	.000
37	....	.490	....	.560	.774
38 40 56	.000	....	....	....	....
40	.511	.528	.543	.590	.818
50	.480	.492	.501	.558	.794
60	.468	.482	.480	.545	.788
70	.445	.468	.480	.530	.780
80	.373	.386	.407	.444	.663
90	0.000	0.000	0.000	0.000	0.000

TABLE 15

INCIDENT SV WAVE IN THE SOLID AGAINST THE WATER: ANGLES OF INCIDENCE AT WHICH THE ENERGY RATIO OF THE REFRACTED P WAVE HAS ITS EXTREME VALUE

r		0.2	0.3	0.4
n	m_1	β_{ext}	β_{ext}	β_{ext}
0.2	1.60	36°12′	36°19′	36°26′
	1.70	.. ..	32 45	.. ..
	1.75	.. ..	31 15	.. ..
0.3	1.60	36 10	36 16	36 23

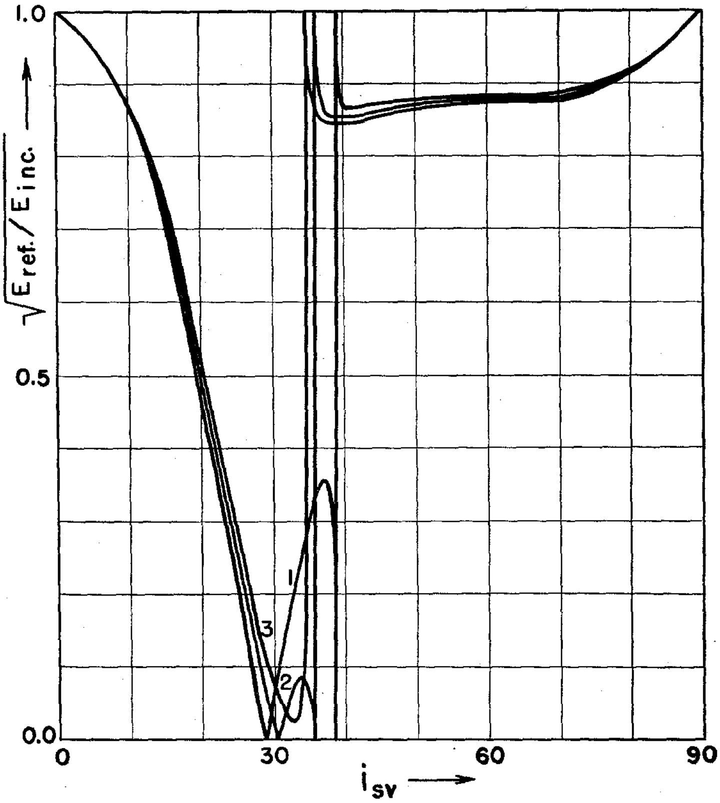

Fig. 7. SV wave incident in the solid against the water. Square root of the energy ratio of the reflected SV wave.

Curve	V_{1p}/V_{1s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.75	0.2	0.3

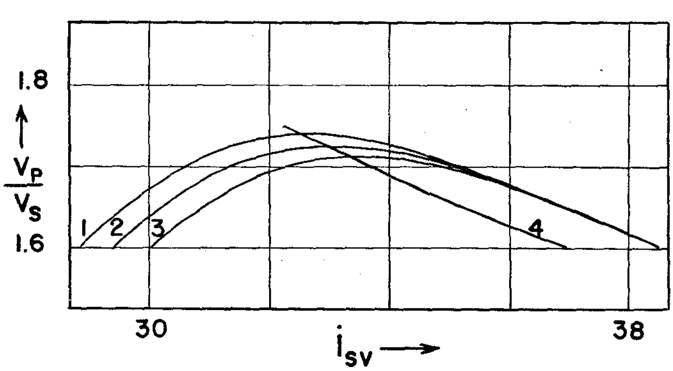

Fig. 8. SV wave incident in the solid against the water.

Curve		V_{2p}/V_{1p}	ρ_2/ρ_1
1	Zeros of the energy ratio of the reflected SV wave	0.2	0.3
2		0.3	0.3
3		0.4	0.3
4	Extreme points of the energy ratio of the reflected SV wave.	0.2	0.3

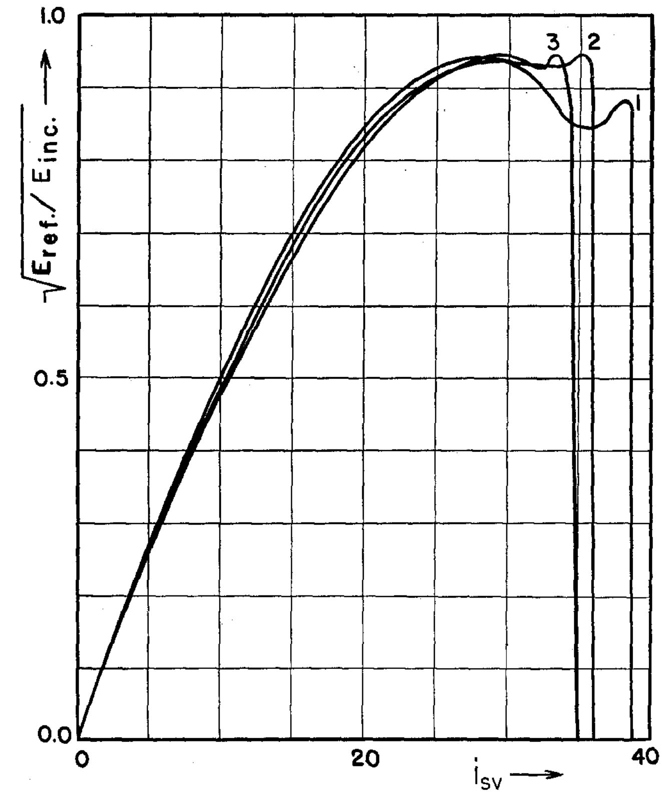

Fig. 9. SV wave incident in the solid against the water. Square root of the energy ratio of the reflected P wave.

Curve	V_{1p}/V_{1s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.75	0.2	0.3

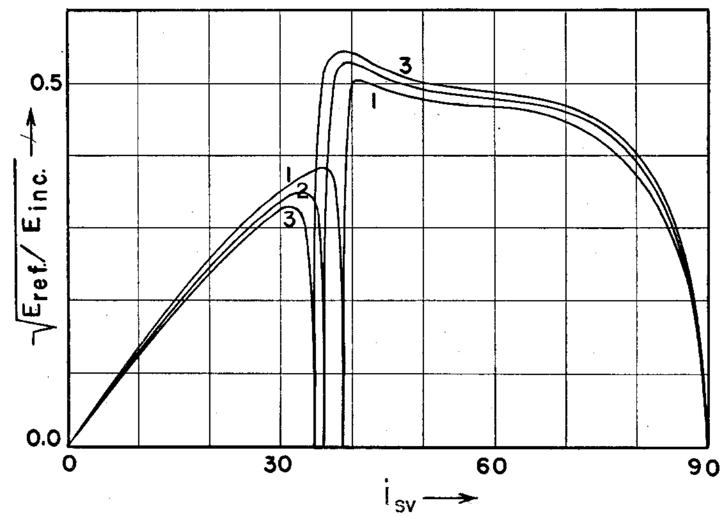

Fig. 10. SV wave incident in the solid against the water. Square root of the energy ratio of the refracted P wave.

Curve	V_{1p}/V_{1s}	V_{2p}/V_{1p}	ρ_2/ρ_1
1	1.6	0.2	0.3
2	1.7	0.2	0.3
3	1.75	0.2	0.3

Summary and Conclusions

General behavior, zeros, and extreme points of different type of waves incident at both sides of the ocean floor, for some values of parameters m, n, and r which are considered to include actual values, are computed; computed values are tabulated and plotted.

The following general conclusions are drawn from the results:

1. Poisson's ratio σ of the solid medium is a dominating factor in the general behavior, zeros, and extreme points of the reflected and refracted waves that travel in the solid and are produced by any possible type of wave incident at either side of the solid-water boundary.

2. n dominates the behavior of refracted waves and also plays an important role in the behavior of all waves around the critical incidence for $\eta = 90°$.

3. General behavior of any wave that is produced by incidence of any type of wave at either side of the boundary is affected very little by changing r, the effect usually being negligibly small.

4. In all cases the values of α_{ext} or β_{ext} change very slightly by changing r. The same statement is true for n except where the occurrence of extreme value is due to a total reflection at $\eta = 90°$.

5. In all cases considered in this paper, peculiar behaviors are observed just before and right after the critical angles of incidence.

6. At a critical angle of incidence the whole energy goes into the reflected wave of the same kind as the incident wave.

7. In all cases considered, most of the energy goes into reflected P or S wave, depending on the angle of incidence. This can be explained by large contrast between densities and velocities in two media. In the case of a P wave incident in the water against the solid, after $\alpha_{c\eta}$ the energy is split between reflected P wave and refracted S wave but still a larger part goes into the reflected P wave.

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